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On the same origin of quantum physics and general relativity from Riemannian geometry and Planck scale formalism

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ABSTRACT

It has been a long time to reconcile quantum physics and general relativity. To date, no globally accepted theory has been proposed to explain all physical observations. In this work, we reformulated the Riemannian geometry in terms of curvature and energy tensors using the Planck scale formalism. The proposed equation can be transformed into Dirac equations in electrodynamic and chromodynamic fields with a reduction in the background curvature. We redefined the mass and charge of leptons in terms of the interactions between the energy of the field and the curvature of the spacetime. The obtained equation is covariant in space–time and invariant with respect to any Planck scale. Therefore, the constants of the universe can be reduced to only two quantities: Planck length and Planck time. We proved that the Einstein field equation from general relativity is actually a relativistic quantum mechanical equation. We further modeled the universe using the equation with Einstein's lambda formalism and found that the universe dynamics could be considered as harmonic oscillators entangled with lambda curvature. This equation can be used to describe the energy transfer between two entangled spacetimes between the same universe and between any two universes (ER=EPR). The singularity of black holes can be avoided at the Planck scale, because space and time are no longer entangled. This equation predicts that information of light from the entangled universe can be transferred to our universe. The gravitational wave background was predicted, and its spectrum was close to that of the observation.

1. Introduction

The unification of the Einstein field equation and quantum theory still lacks a clear and consistent bridge between the two. All theories in the literature require unproven assumptions or inconsistencies. This work bridges the gap between the two theories. In recent years, the James Webb Space Telescope (JWST) has observed several phenomena, including galaxies that had already existed 300 Myr after the big bang, which have never been thought to exist [1,2]. However, our proposed theory suitably explains this phenomenon.

In addition, the discovery of the Big Ring structure challenges the cosmological principle, which posits that the universe should look roughly the same everywhere, on a large scale. The existence of such ultra-large structures like the Big Ring and the Giant Arc questions this fundamental assumption in cosmology. The equation proposed in this study can be used to explain this phenomenon: Our work also explains this finding.

Another conjecture in physics is whether the Einstein-Rosen bridge (ER) and Einstein-Podolsky-Rosen (EPR or entanglement) are physically equivalent. The ER=EPR conjecture awaits rigorous proof [3]. This work also provides further proof of this claim. This work is different from other attempts at unification: (i) string theory, which still lacks experimental observation of extra dimensions [4–6], (ii) loop quantum gravity, which still faces challenges in its compatibility with the Standard Model [7]. In our study, we assume that the new equation should be written in a unitless manner on the Planck scale. Current physical models require at least ten physical constants. Meanwhile, there remain only two constants used in this framework: Planck length and Planck time. In addition, the proposed equation can explain the Gravitational Wave Background (GWB) observed over 15 years by NANOGrav [8].

Applying the Onsager principle on reciprocal relation to the Einstein field equation (EFE), we infer that if a mass can create a curvature (EFE), the curvature can also create a mass. We recap the Ricci tensor before proving each claim in this work. An important concept inferred from the proposed equation is that relaxation of the curvature can create a mass. Because this is a theoretical work, it is organized by topic rather than by

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an ordinary experimental article structure.

The Ricci tensor R_{ij}, stands as a fundamental mathematical form of the curvature inherent to a Riemannian manifold [9,10]. It is derived from the Riemann curvature tensor R^{l}_{ijk} by contraction. This can be expressed as $R_{ij} = R^{k}_{ikj}$ where we take the summation convention over repeated index $\boldsymbol{k}$. The Ricci tensor is symmetric, that is, $R_{ij} = R_{ji}$, and in a local coordinate system, it can be expressed in terms of the metric tensor g_{ij} and its derivatives. The Ricci tensor is used to describe the curvature of space-time in Einstein's field equations and is integral to the study of Einstein manifolds, Ricci flow, and scalar curvature. Although it encapsulates important geometric information, the Ricci tensor does not fully characterize the manifold curvature, a task reserved for the full Riemann curvature tensor. Its utility lies in its simpler structure compared with the Riemann tensor, which facilitates analyses in various geometrical and physical contexts.

1.1. Planck units

In this work, we applied ***Planck length*** and ***Planck time*** to form a Plack-scale invariant equation in addition to the form of a covariant field equation. The ***Planck length***, denoted by l_p, is the smallest measurable unit length. The Planck length embodies this inherent limitation, signifying the smallest possible region in which a particle can be localized, owing to its associated momentum uncertainty. Mathematically, the Planck length is expressed as [11,12]

$$l_p = \sqrt{\frac{G\hbar}{c^3}} \approx 1.6 \times 10^{-35} \text{ meter},$$

where G is the gravitational constant, $\hbar$ is the reduced Planck's constant, and c is the speed of light.

Planck time is intimately linked to Planck length is the ***Planck time***, which is denoted by $\boldsymbol{t_p}$. It represents the smallest measurable unit of time and is derived from the Planck length and the speed of light. Planck time signifies the minimum duration over which any meaningful physical process can occur [1]. Mathematically, the Planck time is expressed as

$$tp = \frac{l_p}{c} \approx 5.3 \times 10^{-44} \text{ seconds}$$

These minuscule quantities are far beyond the reach of our current technology and likely represent the fundamental granular nature of space–time at the quantum level. Understanding these concepts is crucial for effectively applying the unification of gravity and quantum theories, as they represent the scales at which both theories are expected to converge [13]. We propose an equation that is unitless on a Planck scale. The equation can be derived into the Dirac equation, Maxwell's equations, quantum electrodynamics, the Klein-Gordon equation, Einstein Field Equation, and calculation of quark masses as well as neutrino masses (electron, tau, and muon), which are valid compared to the Standard Model [14,15]. The proposed equation could potentially explain the existence of the galaxies observed by the JWST in the early universe (which was not supposed to exist) and the theory's implications on the evolution of the universe.

1.2. Ricci tensor formulation in terms of covariant derivative

From Riemannian Geometry [16], Ricci curvature is defined as the commutator covariant derivative of a vector in a manifold in different directions. We can write the Ricci tensor as:

$$R_{\mu\nu}\psi^A = [D_\mu,\ D_\nu]\psi^A \tag{1}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, ψ^A is a vector in the manifold or spacetime, D_μ is the covariant derivative in the coordinate x^μ in spacetime.

2. The unified equation

According to Einstein Field Equation in General Relativity [9,17]

$$\kappa\varepsilon_{\mu\nu} = R_{\mu\nu} - \frac{g_{\mu\nu}R}{2} \sim R_{\mu\nu}, \tag{2}$$

where $\varepsilon_{\mu\nu}$ denotes the energy tensor. According to the Planck unit, we can rewrite this equation in terms of the Planck length l_p and Planck energy ε_p because $\kappa = 8\pi l_p/\varepsilon_p = 8\pi G/c^4$. We get

$$8\pi l_p \frac{\varepsilon_{\mu\nu}}{\varepsilon_p} = R_{\mu\nu} = [D_\mu,\ D_\nu]$$

or

$$\frac{\varepsilon_{\mu\nu}}{\varepsilon_p l_p} = \frac{l_p^{-2}}{8\pi}[D_\mu,\ D_\nu]. \tag{3}$$

3. Dirac equations from einstein field equations

This section provides mathematical proof that a unified equation implies a Dirac equation with an electromagnetic (EM) gauge with curvature relaxation or inflation. From Eq. (3), since

$$\varepsilon_p l_p = \hbar c$$

and assume Lie group of D_μ over spacetime manifold, we obtain

$$[D_\mu,\ D_\nu] = i\varepsilon^k_{\mu\nu}D_k,$$

where $\varepsilon^k_{\mu\nu}$ is the Levi-Civita tensor [18]. Upon normalizing $D_\mu \rightarrow \frac{l_p^{-1}}{\sqrt{8\pi}}D_u$ to a unitless covariant derivative, we arrive at

$$\frac{\varepsilon_{\mu\nu}}{\hbar c} = [D_\mu,\ D_\nu] = i\varepsilon^k_{\mu\nu}D_k, \tag{4}$$

$$\varepsilon_{\mu\nu} = i\hbar c\cdot\varepsilon^k_{\mu\nu}D_k, \tag{5}$$

or

$$\varepsilon = i\hbar c\gamma^k D_k, \tag{6}$$

where γ^k are Dirac matrices. Finally, we derived the Dirac equation (DE) for a zero-mass object [19,20]. By substituting the wave function $\psi = e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$ into Eq. (6), we obtain

$$\varepsilon = \hbar ck = \frac{hc}{\lambda} = hf,$$

which is the quantum energy of a photon.

Consider Eq. (4) using the definition of $D_\mu = D'_\mu \pm \Gamma'_\mu$, where Γ'_μ are the Christoffel symbols from the curvature of the new spacetime coordinate:

$$[D_\mu,\ D_\nu] = \left[D'_\mu \pm \Gamma'_\mu,\ D'_\mu \pm \Gamma'_\mu\right]$$

$$= [D'_\mu, D'_\nu] \pm D'_\mu\Gamma'_\nu \mp D'_\nu\Gamma'_\mu \pm [\Gamma'_\mu, \Gamma'_\nu] \pm \Gamma'_\mu D'_\nu \mp \Gamma'_\nu D'_\mu.$$

Since

$R'_{\mu\nu} = D'_\mu\Gamma'_\nu - D'_\nu\Gamma'_\mu + \left[\Gamma'_\mu, \Gamma'_\mu\right]$, [21]

$[D_\mu,\ D_\nu] = \left[D'_\mu,\ D'_\nu\right] \pm R'_{\mu\nu} \mp \varepsilon_{\mu\nu}\Gamma'_\mu D'_\nu.$

Because $R'_{\mu\nu}$ in unit of $E/\hbar c$ equal to the LHS, we have

$$R'_{\mu\nu} = \pm\frac{mc^2}{\hbar c}.$$

From Eq. (4), we deduce that

$$\frac{\varepsilon_{\mu\nu}}{\hbar c} = \left[D'_{\mu}, D'_{\nu}\right] \pm \frac{mc^2}{\hbar c} \mp \varepsilon_{\mu\nu}\Gamma'_{\mu}D'_{\nu}.$$

$$\frac{\varepsilon_{\mu\nu}}{\hbar c} = R^0_{\mu\nu} \pm \frac{mc^2}{\hbar c} \mp \varepsilon_{\mu\nu}\Gamma'_{\mu}D'_{\nu}$$

Let $\varepsilon_{\mu\nu}\Gamma'_{\mu} = i\gamma^{\nu}$ and $R^0_{\mu\nu} = \left[D'_{\mu},\ D'_{\nu}\right]$, we arrive at

$$\varepsilon_{\mu\nu} = \hbar c\ R^0_{\mu\nu} \pm mc^2 \mp i\gamma^{\mu}\hbar c D'_{\nu},$$

$$\varepsilon_{\mu\nu} - \hbar c\ R^0_{\mu\nu} = \varepsilon^0_{\mu\nu} = i\hbar c\gamma^{\mu}D'_{\nu} - mc^2, \tag{7}$$

where $\varepsilon^0_{\mu\nu}$ is the effect of energy after resetting the new curvature of spacetime in a new reference frame. Thus, Eq. (7) is the Dirac equation, which is a relativistic quantum theory.

4. The Einstein's special relativity in curved space-time formalism

From the Einstein Field Equation (EFE) in Riemannian geometry [22] and the Planck formalism, we can derive relativistic quantum mechanics equations for fermions. Therefore, there is a corresponding one-to-one mapping from EFE to DE for both zero mass and mass.

We can say that they are the same equations, that we will elaborate further on in the next section.

In conclusion, Eq. (7) could be rewritten as

$$\left(\frac{\varepsilon_{\mu\nu}}{\varepsilon_p}\right)\frac{1}{l_p} = R^0_{\mu\nu} + i\gamma^k D_k - m' \tag{8}$$

where $m' = mc^2/\hbar c$.

The Klein-Gordon equation [23] can be derived from our proposed equation, as follows: Taking the square of Eq. (8), we deduce:

$$\frac{\varepsilon_{\mu\nu}\varepsilon^{\mu\nu}}{\left(\varepsilon_p l_p\right)^2} = R^0_{\mu\nu}R^{0\mu\nu} + \Box^2 + m',$$

$$\frac{\varepsilon^2}{\left(\varepsilon_p l_p\right)^2} = R_0^2 + \Box^2 + m'^2 = R_0^2 + p^2 + m^2. \tag{9}$$

Eq. (9) is the Klein-Gordon equation, which describes bosonic particles with curvature tensors as additional terms:

5. Electromagnetic field and the Maxwell's equations

Upon promoting operator D_{μ} to add A_{μ} vector potential with coupling e to the gauge, c is the velocity of light:

$$D_{\mu} \rightarrow D_{\mu} \pm \frac{ieA_{\mu}}{c}.$$

Introducing the gauge into Eq. (7), we have

$$\frac{\varepsilon_{\mu\nu}}{\varepsilon_p l_p} = \left[D_{\mu},\ D_{\nu}\right] = \left[D^0_{\mu} \pm \frac{ieA_{\mu}}{c},\ D^0_{\nu} \pm \frac{ieA_{\nu}}{c}\right]$$

$$= \left[D^0_{\mu},\ D^0_{\nu}\right] \pm \frac{ie}{c}\left[D_{\mu}A_{\nu} - D_{\nu}A_{\mu}\right] \mp \frac{e^2}{c^2}\left[A_{\nu},\ A_{\mu}\right] \pm \frac{ie}{c}\left[A_{\mu}D_{\nu} - A_{\mu}D_{\nu}\right]$$

$$= R^0_{\mu\nu} \pm \frac{ie}{c}F_{\mu\nu} \pm i\frac{e}{c}E_{\mu\nu}A_{\mu}D_{\nu} \mp \frac{e^2}{c^2}\left[A_{\nu},\ A_{\mu}\right]. \tag{10}$$

5.1. Derivation of Maxwell's equations

Suppose the term in imaginary part cancel each other out and equal to zero.

$$\therefore F_{\mu\nu} = \varepsilon_{\mu\nu}A_{\mu}D_{\nu},$$

$$F_{\mu\nu} = -\frac{e}{l'_p}\cdot\varepsilon_{\mu\nu}x_{\mu}D_{\nu},$$

where $l'_p = \frac{1}{L_{now}} = m_e$.

$$F_{\mu\nu} = -\frac{e\left(x_{\mu}D_{\nu} - x_{\nu}D_{\mu}\right)}{l'_p},$$

$$F_{\mu\nu} = -L_{\mu\nu}\cdot\frac{e}{m_e}.$$

Consider its partial derivative,

$$\frac{\partial F_{\mu\nu}}{\partial x^{\mu}} = \frac{ep_{\nu}}{m_e} = i\hbar\frac{e}{m_e}D_{\nu} = J_{\nu}. \tag{11}$$

Suppose $e = 1/L$, and we proved Maxwell's equations in (11). From (10), with the imaginary term cancelled out as Maxwell's equation and the relation between the electron mass and charge in terms of curvature, we again obtain the Dirac equation.

5.2. Field theory of electromagnetics

$$\frac{\varepsilon^2}{\left(\varepsilon_p l_p\right)^2} = R_0^2 + F^2 + m_e^2 + \Box^2$$

$$= R_0^2 + F^2 + m_e^2 + \left(\frac{e}{c}\right)^2 A_{\mu}A^{\mu} + \Box^2 \tag{12}$$

Next, we obtained the gauge term in EM theory. Taking the square root of Eq. (12), we obtain

$$\frac{\varepsilon_{\mu\nu}}{\varepsilon_p l_p} = R'_{\mu\nu} + \frac{mc^2}{\hbar c} + ir^{\mu}D_{\mu} - \frac{e}{c}A_{\mu} \tag{13}$$

By offsetting the Ricci and energy terms, we obtain the Field Theory of Electromagnetics, which is a differential equation with an additional gauge vector field A_{μ} for electron-photon interactions. The gauge vector was introduced into the following equation:

$$i\gamma^{\mu}D_{\mu} - \frac{e}{c}A_{\mu} + \frac{mc^2}{\hbar c} = 0. \tag{14}$$

6. Chromodynamics: quark and gluon field theory

From the unified equation, we promote $D_{\mu} \rightarrow D_{\mu} + ig_s t_a A^a_{\mu}$, where t_a is $\frac{\lambda_a}{2}$, λ_a is the Gellman matrix, a is the gluon field, and g_s is the coupling constant.

$$\frac{\varepsilon_{\mu\nu}}{\varepsilon_p l_p} = \left[D_{\mu},\ D_{\nu}\right] = \left[D_{\mu} \pm ig_s t_a A^a_{\mu},\ D_{\nu} \pm ig_s t_a A^a_{\nu}\right]$$

$$= \left[D_{\mu},\ D_{\nu}\right] \pm it_a\left(D_{\mu}A^a_{\nu} - D_{\nu}A^a_{\mu}\right) \mp g_s^2 t_a\left[A^a_{\mu},\ A^a_{\nu}\right] + \varepsilon_{\mu\nu}g_s t_a A^a_{\mu}D_{\nu} \tag{15}$$

Let $G^a_{\mu\nu} = D_{\mu}A^a_{\nu} - D_{\nu}A^a_{\mu} \pm ig_s\left[A_{\mu},\ A_{\nu}\right]$ and

$$G_{\mu\nu} = \sum_{a=1}^{8} t_a G^a_{\mu\nu}. \tag{16}$$

In the last term $\left[A_{\mu},\ A_{\nu}\right] = g_s[t_b,\ t_c]A^b_{\mu}A^c_{\nu}$. Then, $[t_a,\ t_c] = if^a_{bc}t_a$ and a, b, $c \in \{1, 2, \ldots, 8\}$ where f^a_{bc} are the structure constants of SU(3) of t_a.

From Eq. (1),

$$R_{\mu\nu} = R^0_{\mu\nu} + g_s G_{\mu\nu} + i\gamma^{\mu}g_s D_{\nu}$$

$$= \left(\frac{mc^2}{\hbar c} + iG_{\mu\nu} + i\gamma^{\mu}D_{\mu}\right)g_s \tag{17}$$

where $\gamma^{\mu}g_s D_{\nu} = \varepsilon^{\mu}_{ab}t_c A^c_a D_b$.

Taking square of (17) and square root back gives

$$\frac{\varepsilon_{\mu\nu}}{\varepsilon_p l_p} = R_{\mu\nu} = g_s\left(-\frac{mc^2}{\hbar c} + i\gamma^\mu\left(D_\mu + ig_s t_a A_\mu^a\right)\right). \tag{18}$$

Eq. (18) is Dirac equation of motion for gluons and quarks.

7. Curvature relaxation and mass generation

The relaxation of R which describes a curvature larger than R^0 results in R^0 with the momentum and mass. Therefore, as the universe expands, it produces both mass and kinetic energies. In other words, the curvature is converted into mass and kinetic energy, known as momentum, electrons, or photons (or quarks and gluons), as discussed in the previous section. Fig. 1 depicts the reciprocal relation (Onsager principle) that mass creates curvature (Einstein's field equation), and that curvature can create mass (this work).

$$\frac{\varepsilon^2}{(\hbar c)^2} = R^2 = R^0_{\mu\nu} + m^2 + p^2 \tag{19}$$

8. Computed quark and neutrino masses vs the standard model

The proposed equation can compute the masses of up and down quarks and neutrinos (electron, muon, and tau), which is consistent with the Standard Model (SM).

8.1. Electron mass and quark masses

We begin with electron and quark mass calculations using the proposed equations, which conform to the values from the Standard Model. Subsequently, we derived the masses of the neutrinos (electrons, muons, and tau). According to Eq. (10), where the last term is the interaction term, we arrive at

$$\frac{e^2}{c^2} = m_e.$$

The charge must be converted into a unit of Coulomb interactions. Thus, we added the terms electric and magnetic field interactions and obtained the correct unit of electron charge.

$$e^2 \rightarrow \frac{e^2}{\sqrt{4\pi\epsilon_0}\cdot\sqrt{\mu_0/4\pi}} = \frac{e^2}{\sqrt{\epsilon_0\mu_0}} = e^2 c$$

Thus

$$\frac{e^2}{c^2}\cdot c = m_e \rightarrow e^2 = m_e\cdot c$$

where $m_e \propto$ *curvature of the universe* $\propto \frac{1}{\textit{radius of the universe}} = 1/L_{now}$.

$$m_e = 9.1 \times 10^{-31}\ \text{kg}.$$

Hence

$$L_{now} \sim \frac{1}{m_e} \sim 10^{30}\ \text{m}.$$

Charge of electron can be calculated by

$$e \sim \sqrt{9 \times 10^{-31} \times \frac{1}{3} \times 10^{-8} \times 4\pi\left(\frac{s}{m}\right)} = \sqrt{\frac{3}{10} \times 4\pi} \times 10^{-19}\ \text{C}$$

$$\sim 1.9 \times 10^{-19}\ \text{C}$$

which is close to actual value of 1.6×10^{-19} C.

In conclusion

$$e^2 = \frac{m_e}{c} = \frac{1}{c\cdot L_{now}}.$$

As the universe expands, the charge and mass of electrons decreases, resulting in less interaction in the aging universe. The electron charge and mass are no longer constant. They depended on the radius of the universe at the time.

We can consider the mass of the quark in terms of the mass of the electron in the same way, considering the charge. For the up and down quarks, the charges were $\frac{2}{3}e$ and $-\frac{1}{3}e$ respectively. Because a quark is equivalent to a confined electron in the fractal dimension of space–time, it results in a larger interaction energy.

The mass of the down quark, m_d should be proportional to $(3e)^2$ because we divide the electron into 1/3 fractal dimensions. Hence,

$$m_d = 9e^2 = 9m_e = 4.5\ \text{MeV},\ (\text{SM value } 4.8\ \text{MeV})$$

the same reason for up quark is for fractal dimension ½. Therefore, its mass

$$m_u = (2e)^2 = 4e^2 = 4m_e = 2\ \text{MeV}\ (\text{SM value } 2.2\ \text{MeV}).$$

9. Mass of leptons (electron, MUON, TAU, with their neutrinos)

The Christoffel symbol Γ_μ is a key component of the local curvature resulting from self-interaction and interaction with the Ricci Tensor (curvature tensor).

- This local curvature results in a mass of particles owing to its local curvature and the surrounding environment.
- If we have two curvature overlaps, according to Gauss's curvature, the new curvature would be the geometric mean of the curvature; for example, curvature K_1 has radius R_1 and curvature K_2 has radius R_2. Combining these yields the curvature

$$K = \frac{1}{R} = \sqrt{K_1 K_2} = \frac{1}{\sqrt{R_1 R_2}}.$$

For this reason, we assume the hierarchy of curvature as in Fig. 2, which illustrates the hierarchical curvature of universe, galaxy, and

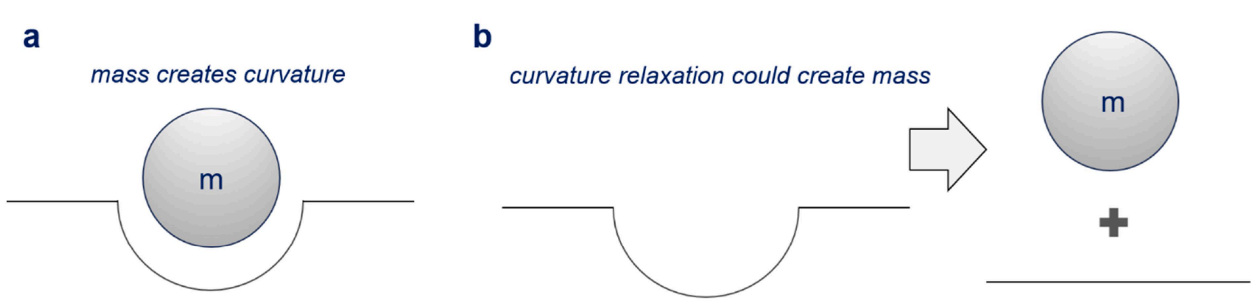

Fig. 1. Onsager principle applied to EFE: (a) mass creates curvature (b) curvature could also create mass.

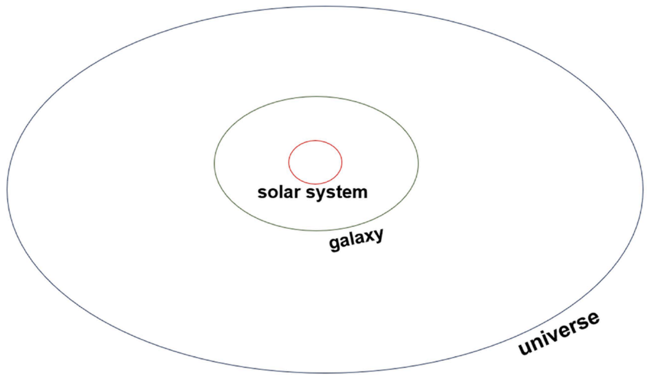

Fig. 2. Hierarchical structure of solar system, galaxy, and universe.

solar system, respectively.

The masses of electrons, muons, and tau can be explained by the different curvatures of universe, galaxy, and solar system, respectively.

We assume R_U, $R_g$and $R_s$stand the radii of universe, galaxy, and solar systems, respectively.

Therefore,

$$m_e = \frac{1}{R_U},\ m_\mu\ (muon\ mass) = \frac{1}{\sqrt{R_U R_g}} m$$

$$m_\tau = \frac{1}{\sqrt{R_s \sqrt{R_U R_g}}}.$$

Rearrange the terms for m_μ and m_τ,

$$m_\mu = \sqrt{\frac{R_U}{R_g} \cdot \frac{1}{R_U^2}} = \sqrt{\frac{R_U}{R_g}} \cdot \frac{1}{R_U} = \sqrt{\frac{R_U}{R_g}} \cdot m_e,$$

$$m_\tau = \sqrt{\frac{\sqrt{R_U R_g}}{R_s} \left(\frac{1}{\sqrt{R_U R_g}} \right)^2}$$

$$= \sqrt{\frac{1}{R_s R_U R_g} \cdot \sqrt{R_U R_g}}$$

$$= \sqrt{\frac{R_g}{R_s} \cdot \frac{R_U}{R_g} \cdot \frac{1}{R_U} \left(\sqrt{\frac{R_U}{R_g}} \cdot \frac{1}{R_U} \right)}$$

$$= \frac{1}{R_u} \cdot \sqrt{\frac{R_g}{R_s} \left(\sqrt{\frac{R_u}{R_g}} \right)^2} = \sqrt{\frac{R_g}{R_s}} \cdot \sqrt{\frac{R_U}{R_g}} \cdot \frac{1}{R_u}$$

$$m_\tau = \sqrt{\frac{R_g}{R_s} \cdot \frac{m_\mu}{m_e}} \cdot m_e = \sqrt{\frac{R_g}{R_s} \cdot \frac{m_\mu}{m_e}} \cdot m_e = \sqrt{\frac{R_g}{R_s}} \cdot m_\mu$$

From the Table 2, we get $\frac{m_\mu}{m_e} = \sqrt{\frac{R_U}{R_g}} \approx 212$ and

$$\frac{m_\tau}{m_\mu} = \sqrt{\frac{R_g}{R_s}} \approx 17.7$$

These parameters were used to calculate the mass of their neutrino counterparts, m_{ν_e}, m_{ν_μ}, and m_{ν_τ} or the mass of (electron, muon, and tau) neutrinos, respectively. Assume m_{ν_e} is the mass where the maximum radius of the universe takes place.

$$\therefore \frac{m_{\nu_e}}{m_e} = \frac{\frac{1}{R_{max}}}{\frac{1}{R_{now}}} = \frac{\mathrm{R_{now}}}{\mathrm{R_{max}}} = \frac{10^{30}}{10^{35}}$$

$$R_{max} = \frac{1}{l_p} \approx 10^{-5}$$

$$m_{\nu_e} = \tfrac{10^{-5}}{2} \times\ 0.5\ \mathrm{MeV/c^2} = 2.5\ \mathrm{eV}\ (\text{Standard Model: } <2.2\ \mathrm{eV})$$

$$m_{\nu_\mu} = \sqrt{\frac{R_{max}}{R_g}} \cdot m_{\nu_e} = \sqrt{\frac{R_{max}}{R_{now}}} \cdot \sqrt{\frac{R_{now}}{R_g}} \cdot m_{\nu_e}$$

$$= 10^{\frac{5}{2}} \cdot 212 \cdot 2.5 = 0.168\ \mathrm{MeV/c^2}\ (\text{SM value } <1.7\ \mathrm{MeV})$$

$$m_{\nu_\tau} = \sqrt{\tfrac{R_g}{R_s}} m_{\nu_\mu} = 3\ \mathrm{MeV/c^2}\ (\text{SM value } <15.5\ \mathrm{GeV/c^2})$$

We wrap up the masses calculated from our theory and compare them with the Standard model as follows (Table 1).

The occurrence of neutrino oscillations could possibly be caused by uncertainty in the hierarchies of the curvature. The charges of the neutrinos disappear because they are fermion particles in the universe with zero curvature (radius is infinity; $E^2 = mc = c/L$, L approaches infinity as E approaches zero).

10. Universe dynamics

From the unified equation and modified EFE: $\kappa \varepsilon_{\mu\nu} = R_{\mu\nu} - \frac{g^\lambda_{\mu\nu} R}{2} + \Lambda g_{\mu\nu}$, let $\Lambda g_{\mu\nu} = \Lambda_{\mu\nu}$ is another curvature field conjugated to $R_{\mu\nu}$ with covariant derivatives D^λ_ν and metric tensor $g^\lambda_{\mu\nu}$, we modify the below equation coupling to the new $\Lambda_{\mu\nu}$ field with

$$R_{\mu\nu} = [D_\mu,\ D_\nu],$$

$$\frac{i}{2} [D_\mu,\ D^\lambda_\nu] = \frac{g^\lambda_{\mu\nu} R}{2}.$$

Suppose $\Lambda g_{\mu\nu} = \Lambda_{\mu\nu}$ term is another field of curvature we could rewrite

$$\frac{\varepsilon_{\mu\nu}}{\varepsilon_p} = l_p \{ [D_\mu,\ D_\nu] + \Lambda_{\mu\nu} \} + \frac{i}{2} [D_\mu,\ D^\lambda_\nu]. \tag{20}$$

Squaring the above equation gives

$$\frac{\varepsilon^2}{e_p^2} = l_p^2 \{ R^2 + \Lambda^2 - [\Lambda,\ R] \}.$$

Let $a\dagger = R + i\Lambda$ and $a = R - i\Lambda$ and for simplicity let $l_p R \to R$ and $l_p \Lambda \to \Lambda$, now we obtain

$$\left(\frac{\varepsilon}{\varepsilon_p} \right)^2 = a^\dagger a - [\Lambda,\ R]. \tag{21}$$

Define

$$[\Lambda,\ R] \sim \frac{i}{2} [D_\mu,\ D^\lambda_\nu] = \frac{1}{2}. \tag{22}$$

Table 1
Summarized table of masses computed from the proposed equation (Green) and the Standard Model (Blue).

Types	Prediction	Standard Model
Quarks	**U**	**U**
	2MeV	2.2 MeV
	D	**D**
	4.5MeV	4.7 MeV
Neutrinos	$\boldsymbol{\nu_e}$	$\boldsymbol{\nu_e}$
	2.5 eV	< 2.2 eV
	$\boldsymbol{\nu_\mu}$	$\boldsymbol{\nu_\mu}$
	0.168 MeV	<1.7 MeV
	$\boldsymbol{\nu_\tau}$	$\boldsymbol{\nu_\tau}$
	3 MeV	< 15.5 GeV

Table 2
Masses of electron, muon and tau from the Standard Model. This will be used to compute the ratio in relation to the solar system/galaxy/universe radii [26–29].

	eV/c^2
m_e	0.5 M
m_μ	105.7 M
m_τ	1.77 G

10.1. The unified equation as harmonic oscillator of universe curvature

From Eq. (22), we can deduce that $\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 = a^\dagger a + \frac{1}{2}$. Substituting the term l_p back, we arrive at:

$$\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 = l_p\left(a^\dagger a + \frac{1}{2}\right)$$

where $[D_\mu, D_\nu^\lambda] = -i$ since R and Λ are conjugated.

$$\varepsilon^2 = (\varepsilon_p l_p)^2\left(a^\dagger a + \frac{1}{2}\right)$$

thus

$$\varepsilon_N = \varepsilon_p l_p\sqrt{a^\dagger a + \frac{1}{2}}$$

or

$$\varepsilon_N = \hbar c\sqrt{N + 1/2}$$

The energy of the universe with a lambda conjugate curvature is quantized in a manner similar to that of a harmonic oscillator. However, the term $\sqrt{N + \frac{1}{2}}$ at the lowest energy is $\varepsilon_0 = \frac{\hbar c}{\sqrt{2}} = \frac{\varepsilon_p l_p}{\sqrt{2}}$.

This corresponds to the maximum radius of curvature

$$L_{\max} = \frac{\sqrt{2}}{l_p} = \frac{\sqrt{2}}{1.6} \times 10^{35} \sim 0.9 \times 10^{35}\ \mathrm{m} \sim 10^{35}\ \mathrm{m}.$$

Consider the maximum energy ε_{max}, where the universe is highly dense and has a size of the Planck scale l_p.

$$\frac{\varepsilon}{\varepsilon_p} = l_p\sqrt{N + \frac{1}{2}} \le \frac{\varepsilon_{max}}{\varepsilon_p} = \frac{1}{l_p} \tag{23}$$

Then

$$\sqrt{N + \frac{1}{2}} \le \frac{1}{l_p^2},$$

$$N + \frac{1}{2} \le \frac{1}{l_p^4},$$

$$N_{max} \le \frac{1}{l_p^4} = \frac{R_{max}^4}{2}.$$

10.2. Universe radius plot vs time

We approximate the squared of Eq. (20) by letting the term $[\Lambda, R] = 0$. Now we have

$$\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 = R^2 + \Lambda^2.$$

Assume the R & Λ field oscillates with frequency (ω) with the proper time τ

$$R^2 = l_p^2 \sin^2\omega\tau$$

$$\Lambda^2 = l_p^2 \cos^2\omega\tau$$

$$\therefore L_R(\tau) \sim \frac{1}{l_p|\sin\omega\tau|} \text{ such that } l_p < L_R < \frac{1}{l_p}$$

The radii of the universe are denoted by L_R, L_Λ. The fields $R^2(\tau)$ and Λ^2 are plotted in Fig. 3.

The **maximum age of the universe** can be calculated as

$$T_{max} \sim \frac{R_{max}}{H} = 10^{35} \times \frac{13.8}{1.6} \times 10^9 = 8.6 \times 10^{41} \text{ years}$$

where H is the Hubble constant.

$$f = \frac{H}{R_{max}} = H \cdot l_p \text{ and } \omega = 2\pi H l_p$$

Hence, there should be an upper bound and a lower bound of the universe's radius, which are $\frac{\sqrt{2}}{l_p} \approx \frac{1}{l_p}$ and l_p.

At Planck scale l_p, the eigensolution to the unified equation is

$$\frac{\varepsilon_{\mu\nu}}{\varepsilon_p}\phi = l_p[D_\mu, D_\nu]\phi,$$

where

$$\phi = \sum_i \phi_1^i(x_\mu)\phi_2^i(x_\nu) - \phi_1^i(x_\nu)\phi_2^i(x_\mu)$$

which are entangled wavefunctions.

11. Planck scale and the space-time entanglement

In Planck scale, it is entangled wavefunction between space and time, which is

$$|\phi\rangle = |ct\rangle_1|x\rangle_2 - |x\rangle_1|ct\rangle_2.$$

Therefore, singularity at the Planck scale should not occur from the collapsing black hole (BH) because the operation on $|\phi\rangle$ for collapsing spacetime is a permutation of space and time.

Hence,

$$|\phi\rangle' = |x\rangle_1|ct\rangle_2 - |ct\rangle_1|x\rangle_2 = -|\phi\rangle$$

resulting in the reversal of the sign of the wavefunction, and thus the expansion of the newborn universe or the so-called white hole (WH).

The entangled $|\phi\rangle$ for the BH state implies that as t increases, x decreases at the Planck scale with a toggle between x and ct. Therefore, as t increases, x increases in the WH state $|\phi\rangle'$.

12. Dynamics of black hole

White holes (WH) are related to black hole (BH) dynamics. In the case of a collapsing BH at the Planck scale, the energy of expansion should be proportional to its curvature $1/l_p$ which is the constant explosion energy for every universe or WH.

$$\frac{\varepsilon_{\mu\nu}}{\varepsilon_p} = l_p[D_\mu, D_\nu] = \gamma^k l_p D_k \tag{24}$$

Squaring the Eq. (24) gives $\frac{\varepsilon_0^2}{\varepsilon_p^2} = l_p^2(D_R^2 \pm D_{ct^2}^2)$

Suppose $\phi = \phi_0 e^{j\vec{k}_R\cdot\vec{R}\pm\omega t}$,

$$\left(\frac{\varepsilon_0}{\varepsilon_p}\right)^2 = l_p^2\left(k_R^2 \pm \left(\frac{\omega}{c}\right)^2\right)$$

Let $k_R = \frac{2\pi}{L} = \frac{1}{L}$ and $\omega = \frac{2\pi}{T} = \frac{1}{T}$.

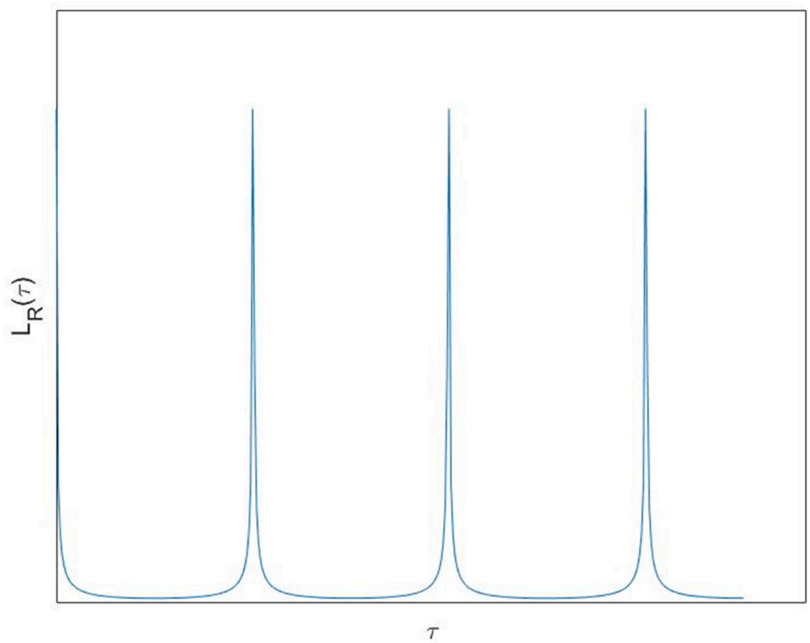

Fig. 3. Universe radius was plotted vs time. The maximum radius would be inversely proportional to the Planck length ($\frac{1}{l_p}$). $L_R(\tau) \sim \frac{1}{l_p|\sin\omega\tau|}$.

$$\bar{\varepsilon}_0^2 = l_p^2\left(\frac{1}{\bar{L}^2} \pm \frac{1}{(c\bar{T})^2}\right).$$

Rewrite this equation to $\bar{L}$,

$$\frac{\bar{\varepsilon}_0^2}{l_p^2} \pm \frac{1}{(c\bar{T})^2} = \frac{1}{\bar{L}^2},$$

$$R(t) = \pm\sqrt{c^2t^2 \pm R_{max}^2}$$

$$\bar{L} = \pm\frac{c\bar{T}}{\sqrt{1 \pm \frac{\varepsilon_0 (c\bar{T})^2}{\varepsilon_p l_p^2}}} \text{ and } ct = \frac{\pm R(t)}{\sqrt{1 \pm \left(R_{max}^2/c^2t^2\right)}}$$

Let $\bar{T} = T\sin\omega\tau$,

$$\bar{L} = \frac{c\bar{T}}{\sqrt{1 - \frac{\varepsilon_0}{\varepsilon_p}.\frac{(c\bar{T})^2}{l_p^2}}}$$

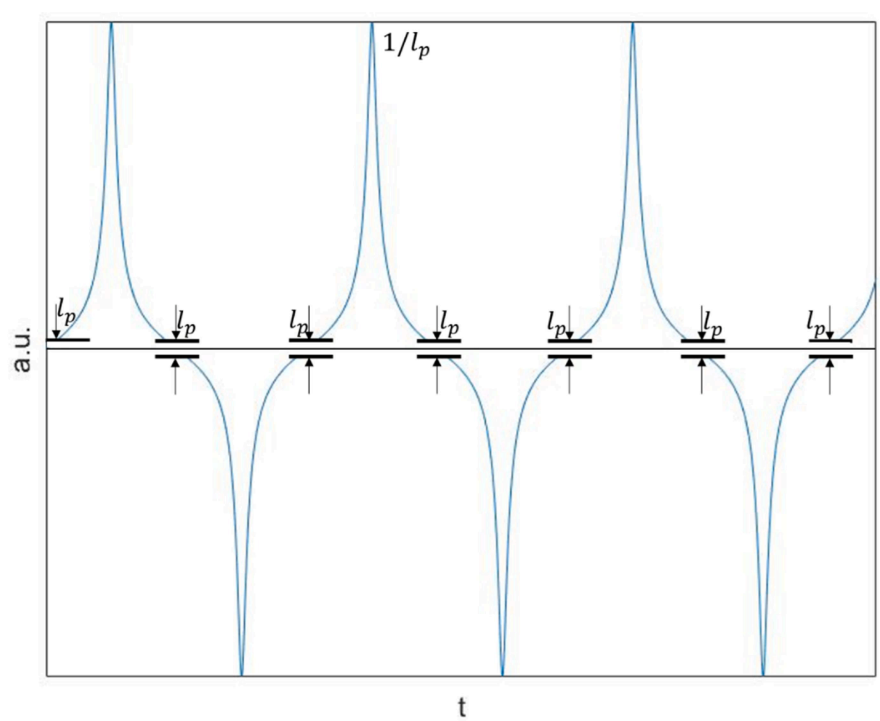

Fig. 4. The universe radius cannot collapse below Plank length (l_p) and the maximum expansion be *1/l_p*.

The negative curvature of the BH results in a positive curvature of the WH and expansion with energy in the Planck scale. Fig. 4 shows the universe radius expansion and collapse with the lower limit of Planck length.

$$\frac{\varepsilon_p l_p^2}{\varepsilon_0} = R_{max} = \frac{1}{l_p^2},$$

$$\varepsilon_p l_p^4 = \varepsilon_0$$

13. ER = EPR

The conjecture that ER=EPR can be proved in our formulation (Fig. 5). Before we discuss whether ER=EPR, we consider the space-time entanglement between a universe and its conjugate.

13.1. Space-time entanglement between universe and conjugated universe $\Lambda_{\mu\nu}$

Since $\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 = R^2 + \Lambda^2,$

R^2 could be relaxed and become $R_0^2 + m^2 + p^2$.

Then $\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 = R_0^2 + m^2 + p^2 + \Lambda^2$. $\Lambda_{\mu\nu}$ can absorb the momentum and mass terms, increase its curvature, and turn into $\Lambda'_{\mu\nu}$

$$\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 = R_0^2 + \Lambda'^2$$

$\Lambda'_{\mu\nu}$ could then be relaxed and release mass in its universe, then

$$\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 = \varepsilon_0^2 + \Lambda^2 + m^2 + p^2.$$

Therefore, if two universes are entangled, we can send an energy of $\varepsilon_0^2 = m^2 + p^2$ to another universe via curvature encoding or holographic coding. Next, we consider the cases of black hole and white hole.

13.2. BH & WH entangled with no singularity

As discussed earlier, singularity from collapsing the BH can be avoided at the Planck scale by space-time inversion. Suppose that the wave function of the BH can be described by

$$e^{-\alpha R}\phi_0 = \phi.$$

where $\alpha(t)$ is the scaling factor for ϕ_0 and increase with time. Consider

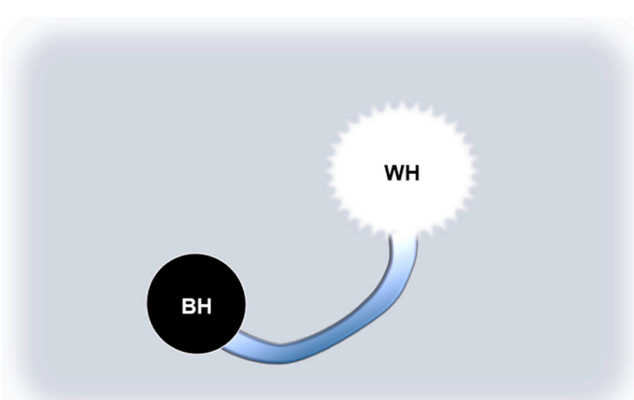

Fig. 5. Blackhole and white hole entanglement.

$$\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 \phi = l_p^2 [\Box^2 + m^2]\phi$$

where ϕ is eigen wave function for space & time.

$$\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 \phi = l_p^2 [\partial_{ct}^2 - \partial_R^2] e^{-\alpha R}\phi_0$$

$$= l_p^2 \left[\frac{R^2}{c^2}\ddot{\alpha} - \alpha^2\right]\phi$$

$$\therefore \left(\frac{\varepsilon}{\varepsilon_p}\right)^2 \frac{1}{l_p^2} = \left(\frac{R}{c}\right)^2 \ddot{\alpha} - \alpha^2$$

Plug this α back to $e^{-\alpha R}\phi_0$, therefore at $t = \frac{1}{cl_p}\left(\frac{\varepsilon}{\varepsilon_p}\right)$ it starts to collapse to BH with radius (R):

$$R = ct = \frac{1}{l_p}\left(\frac{\varepsilon}{\varepsilon_p}\right).$$

Rescaling l_p^2 we obtain

$$R_S = l_p\left(\frac{\varepsilon}{\varepsilon_p}\right).$$

Let $\varepsilon = Mc^2$,

$$R_S = \frac{G}{c^4}Mc^2 = \frac{GM}{c^2}.$$

Now we derived R_S as the Schwarzschild radius.

As the BH collapses and reach l_p, Planck-scale eigenfunction change its sign.

$$|\phi_0\rangle = |ct_1\rangle|x_2\rangle - |ct_2\rangle|x_1\rangle,$$

$$BH|\phi_0\rangle = |ct_2\rangle|x_1\rangle - |ct_1\rangle|x_2\rangle.$$

Therefore, it stops collapse and start to expand instead due to $|x\rangle$ and $|ct\rangle$ interchange. This results in a newborn WH or universe.

Because BH and WH are entangled and conjugated, we can write the unified equation as

$$\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 = l_p^2(R^2 + \Lambda^2)$$

Hence, energy transfer from particles falling into the BH can be transferred to the WH or conjugated universe via the same scenario, similar to the case of lambda. Experimental energy transfer via entanglement was reported by Gómez-Ruiz [24].

This could solve the *information loss paradox* in BH, as JWST can see light and galaxies before a big bang from the conjugated universe.

EPR = ER in the same universe

In this case, we have to rewrite the unified equations to

$$\frac{\varepsilon_{\mu\nu}^{12}}{\varepsilon_p} = l_p\left[\partial_\mu^1, \partial_\nu^2\right] + l_p\left[\partial_\mu^2, \partial_\nu^1\right] + l_p\left[\partial_\mu^2, \partial_\nu^2\right] \quad (25)$$

14. Eigensolution

The eigensolution is therefore

$$\phi(x_\mu, x_\nu) = \phi_1(x_\mu)\phi_2(x_\nu).$$

which is an entangled state. Upon squaring (25),

$$\left(\frac{\varepsilon^{12}}{\varepsilon_p}\right)^2 = R_1^2 + R_2^2.$$

We used the same method to transport energy between entangled spaces R_1 and R_2 in the same universe [24].

15. Gravity in EM field theory

We promote D_μ to $D_\mu + \Gamma_\mu + eA_\mu$. The equation becomes

$$\frac{\varepsilon_{\mu\nu}}{\varepsilon_p l_p'} = \left[D_\mu + \Gamma_\mu + eA_\mu,\ D_\nu + \Gamma_\mu + eA_\mu\right]$$

where Γ_μ denotes the Christoffel symbol. From the above equation, we use the same method as Γ_μ promotes to $\Gamma_\mu + eA_\mu$.

Therefore

$$\frac{\varepsilon_{\mu\nu}}{\varepsilon_p l_p} = \left[D_\mu, D_\nu\right] + \gamma^\mu\left(D_\mu + i\Gamma_\mu + eA_\mu\right)$$

$$= R_0 - mc^2 + i\gamma^\mu\left(D_\mu + \Gamma_\mu + ieA_\mu\right)$$

$$= R_0 - mc^2 + i\gamma^\mu D_\mu$$

Thus, $D_\mu \to D_\mu + \Gamma_\mu$ and $\Gamma_\mu \to \Gamma_\mu + ieA_\mu$.

Finally, we obtain the Dirac Equation with gravitational term Γ_μ included. The term Γ_μ absorbs EM and quark-gluon interactions and creates a local curvature and hence the mass of the particles.

Γ_μ could be written as $\frac{1}{r}$ as its unit is $[1/L]$ where

$$\frac{1}{r} = 4\pi \int_S \frac{e^{-ikx}}{k^2} d^3k.$$

This is similar to a gauge with no charge, in which the mass cancels out the term eA_μ to consort into the local curvature for EM interaction. Therefore, we can calculate the mass directly from the interaction term.

16. Solution to the unified equation with lambda formalism

The R & Λ terms can be considered as the oscillating energy between R & Λ where the lower and upper bounds are $\frac{1}{l_p}$ and l_p.

$$\frac{\varepsilon^2}{\varepsilon_p^2} = l_p^2\left(R^2 + \Lambda^2\right)$$

$$= l_p^2\left(\cos^2\omega\tau + \sin^2\omega\tau\right) = l_p^2$$

where τ is proper time. Thus

$$R = l_p \cos\omega\tau$$

and universe radius is simply

$$L = \frac{1}{l_p \cos\omega\tau} = \frac{R_{max}}{\cos\omega\tau}.$$

The evolution of the radius of the universe according to R^2, Λ^2, L_R and L_Λ versus time is plotted in Fig. 6.

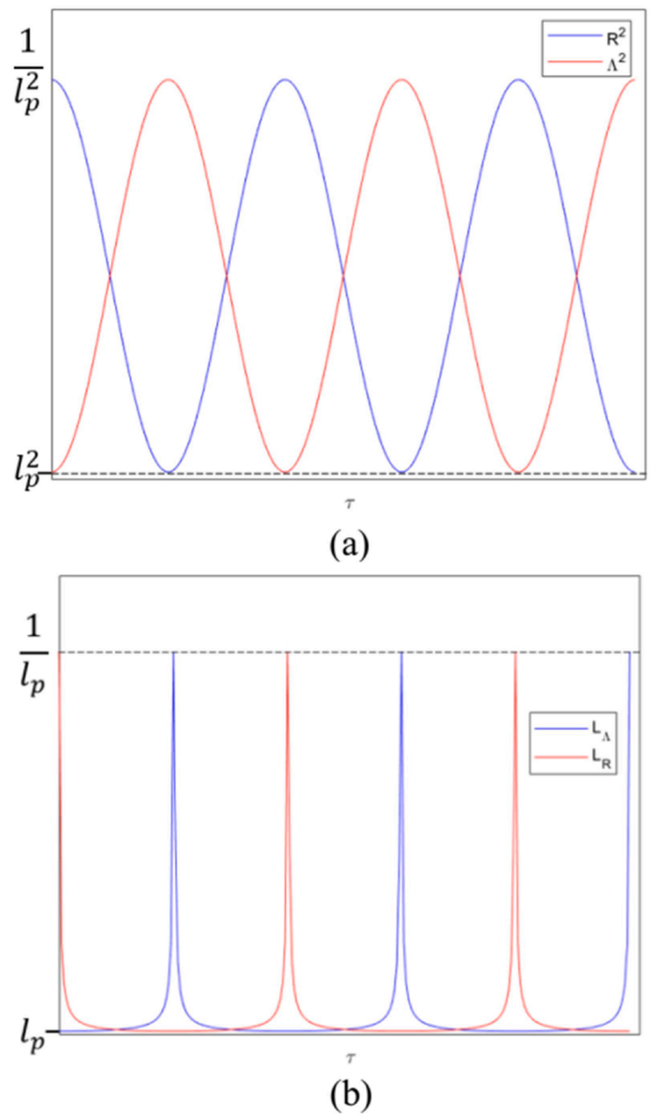

Fig. 6. Solution to proposed equation with lambda formalism, plotted against time τ. (a) $R^2$and Λ^2 versus time and (b) L_R and L_Λ versus time.

17. Gravitational wave background and big ring structure

For each E_n, there exists eigen wave function which contain oscillation of curvature with frequency f_n

Since $E_n = \sqrt{n + \frac{1}{2}} \cdot \hbar c l_p = \frac{c l_p}{\sqrt{n + \frac{1}{2}}}\hbar$. Let $f_n = \frac{C}{R_{max}} \cdot \frac{1}{\sqrt{n + \frac{1}{2}}} = \frac{c l_p}{\sqrt{n + \frac{1}{2}}}$.

We can rewrite E_n as

$$E_n = \frac{\left(c l_p\right)^2 \hbar}{\sqrt{n + 1/2} \cdot c l_p} = \frac{\hbar\left(c l_p\right)^2}{f_n}.$$

Density of state in 3D is $\rho(E) = E^{\frac{1}{2}}$,

$$N(f) = \int \rho(E) dE = \int E^{\frac{1}{2}} dE = \frac{\kappa E^{\frac{3}{2}}}{3/2}.$$

$$N(f_n) = \frac{2}{3}\kappa \cdot \frac{\hbar^{\frac{3}{2}}\left(c l_p\right)^3}{f_n^{\frac{3}{2}}} = \frac{\frac{2}{3}\kappa \hbar^{\frac{3}{2}}\left(c l_p\right)^3 \left(n + \frac{1}{2}\right)^{\frac{1}{2}\cdot\frac{3}{2}}}{\left(c l_p\right)^{\frac{3}{2}}}$$

$$N(f_n) = \frac{2}{3}\kappa \hbar^{\frac{3}{2}}\left(c l_p\left(n + \frac{1}{2}\right)\right)^{3/2} \propto \frac{1}{f^{\frac{3}{2}}}$$

where N is the quantum number of gravitational wave. These power laws correspond to the current discovery of the gravitational background. Our prediction yielded $N^2 \propto f^{-3}$ which is close to $N^2 \propto f^{-4.3}$ reported by [8].

After relaxation of the curvature, it releases momentum and mass. It is also possible to release angular momentum. Thus,

$$E^2 = R^2 + p^2 + m^2 = R^2 + \frac{L^2}{r^2} + m^2. \tag{26}$$

A structure with ring radius r should carry angular momentum L, as the above equation. Stars at greater distances from the center of a galaxy attain higher velocities than those near the center of the galaxy. This can be explained by Eq. (26), where the angular momentum increases as r increases.

18. Discussion

18.1. Entangled space-time

According to the unified equation, we consider space and time component:

$$[\Delta_x,\ \Delta_t] = \frac{\varepsilon}{\varepsilon_p},$$

$$(\Delta_x\Delta_t - \Delta_t\Delta_x)\{\phi_1(x)\phi_2(t) - \phi_2(t)\phi_1(x)\}.$$

Let the minimal physical meaning length and time be l_p and t_p, then

$$\{\phi_1(x+l_p) - \phi_1(x)\}\{\phi_2(t+t_p) - \phi_2(t)\} - \{\phi_2(x+l_p) - \phi_2(x)\}\{\phi_1(t+t_p) - \phi_1(t)\}$$

$$= \{\phi_1(1)\overline{\phi}_2(1) - \phi_1(0)\overline{\phi}_2(1) + \phi_1(0)\overline{\phi}_2(0) - \phi_1(1)\overline{\phi}_2(0)\}$$

$$- \{\overline{\phi}_1(1)\phi_2(1) - \overline{\phi}_1(0)\phi_2(1) + \overline{\phi}_1(0)\phi_2(0) - \overline{\phi}_1(1)\phi_2(0)\},$$

where, for simplicity, (1) either represents $x + x_p$ or $t + t_p$and (0) represents either x or t respectively, depending on whether the term is for time or space. The bar ($\overline{\phi}$) denotes time and ϕ denotes the space states. The state can then be written as

$$(|1\overline{1}\rangle - |\overline{1}1\rangle) + (|0\overline{1}\rangle - |1\overline{0}\rangle) + (|0\overline{0}\rangle - |\overline{0}0\rangle) - (|1\overline{0}\rangle - |\overline{1}0\rangle).$$

The step-by-step creation of particles using space and time entanglement is illustrated in Fig. 7. This process occurs iteratively as the plank time and length increase during inflation.

18.2. Why universes cannot collide?

According to bosonic unified equation,

$$\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 = R^2 + p^2 + m^2.$$

Assume that wave function $|b\rangle = e^{\frac{iL\phi}{\hbar}}|b_0\rangle$, then

$$\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 |b\rangle = (R^2 + p^2 + m^2)|b\rangle.$$

Square root of the equation above yields

$$\left(\frac{\varepsilon}{\varepsilon_p}\right)\sqrt{|b\rangle} = \left(R + \gamma^\mu p_\mu - m\right)\sqrt{|b\rangle},$$

$$|b\rangle = e^{\frac{iL\phi}{2\hbar}}|b_0\rangle = e^{\frac{iL\phi}{2}}|f_0\rangle,$$

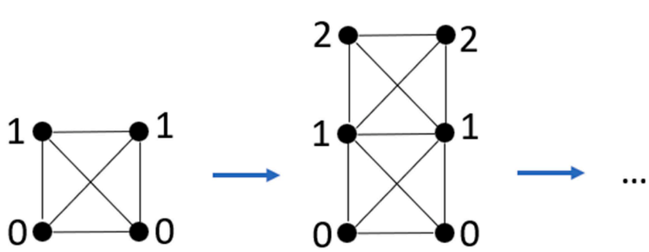

Fig. 7. Explanation of the inflation using iterations of space-time entanglement in Plank scales.

where $|f_0\rangle$ is fermion wave function. Therefore, the universe should carry half-integer spin and are all fermionic. Hence, the Pauli exclusion principle rules out the possibility of collision between universes.

18.3. Increasing velocity of objects far away from a galaxy

Because $\left(\frac{\varepsilon}{\varepsilon_p}\right)^2 = R^2 + \frac{L^2}{r^2} + m^2$ is constant, L is constant if r is the radius of the galaxy.

$$L = m\omega r^2 = I\omega,$$

and ω must be constant. Therefore, $v_x = \omega r_x$.

The velocity of the objects increases as their distance from the center of the galaxy increases because ω is constant. However, it should saturate when r_x approaches r because the curvature of the galaxy vanishes, including the screening effect.

$$m_x = m_0 e^{-\frac{x}{r}}$$

$$L = m_0 e^{\frac{x}{r}} \cdot \omega r^2,$$

$\omega \propto e^{-\frac{x}{r}} \Rightarrow v_x = e^{-\frac{x}{r}} \cdot x$, where $x < r$. These are illustrated in Fig. 8.

It is important to note that our proposed theory matches the galaxy rotation curves [25].

- Galaxies rotate **faster than expected**, especially for distant stars, and usually assume the existence of **dark matter,** a hypothetical matter.
- Our derivation complies with the experimental observation that as the distance (from the galaxy center) increases, the velocity of the stars increases without any additional assumptions. This is an obvious implication from Eq. (26): $E^2 = R^2 + p^2 + m^2 = R^2 + \frac{L^2}{r^2} + m^2$ that as r increases, the angular momentum (L) also increases.

18.4. Two possibilities

As predicted by the proposed equation, the evolution of universes over time can be divided into two distinct schemes, as shown in Fig. 9.

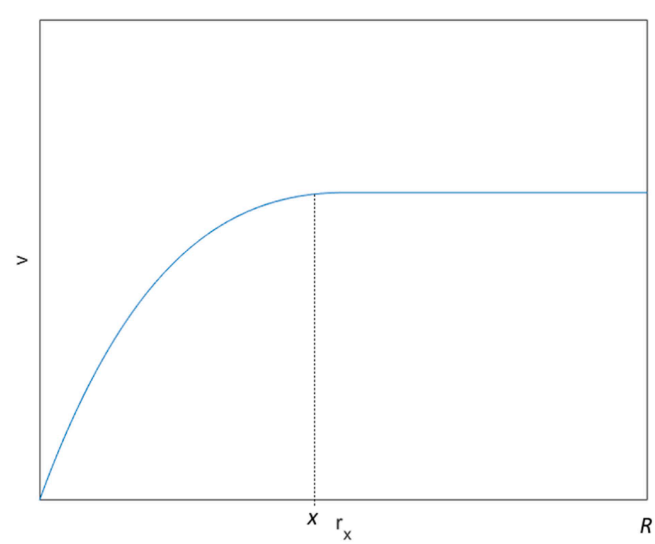

Fig. 8. Velocity plot against the radius **Previously, the galaxy rotation curves** do not match the predictions based solely on the distribution of visible matter.

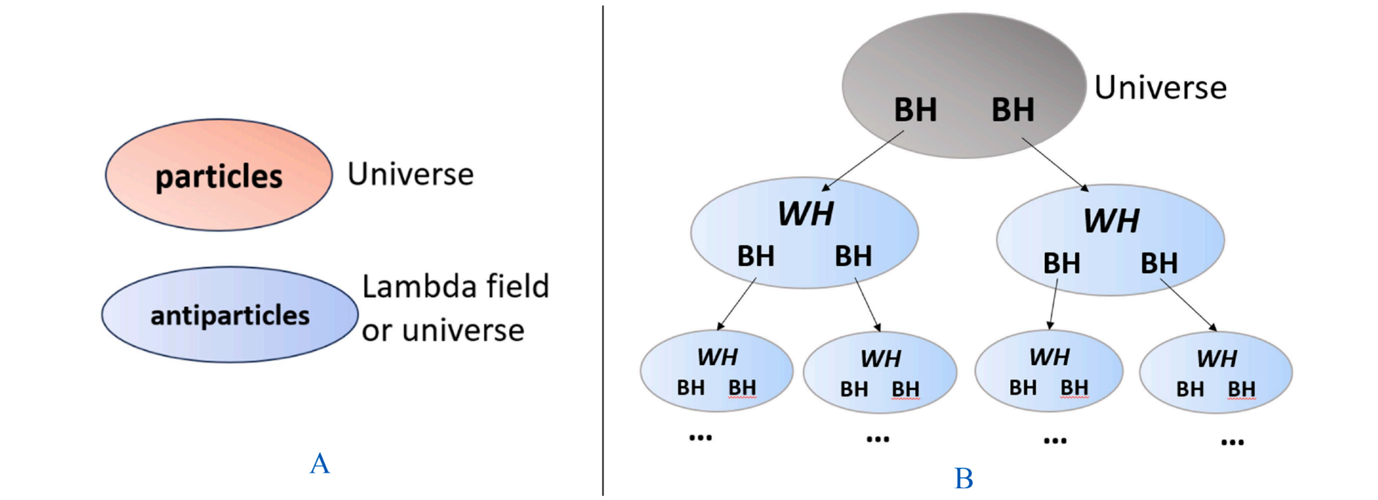

Fig. 9. Two possible cases for the evolution of the universe: (A) it is possible that relative to our universe, the lambda universe contains all antiparticles and travels back in time; (B) it is possible that there will be three of the universes forming from previous blackholes and hence resulting in a multiverse.

19. Conclusion

From Riemannian geometry, we can prove that the Einstein Field Equation (EFE) is equal to the quantum equation by formulating in the Planck-scale unit. In addition to Einstein's principle of covariance that any equation should be independent of a frame of reference, we add an additional requirement that the equation should also be unitless on the Planck scale. The equation shows that relaxation of the curvature induces the creation of mass and (angular) momentum. As the universe expands, the mass increases and finally reaches the point at which it stops expanding. We calculated the masses of the three flavor neutrinos as well as some quarks, owing to the hierarchical curvature and fractal dimension of the quarks. The masses of up/down quarks and masses of (electron, tau, and muon) neutrinos calculated using the proposed equation are consistent with the values from the Standard Model (Table I). At the smallest Planck scale, space and time are entangled and will toggle via a black hole collapsing and avoiding singularity, but instead become expanding space-time. There are two mechanisms by which the universe can form: (i) directly from the lambda field and (ii) from the previous black hole (Fig. 9).

Miscellaneous: *fine structure might not be constant*

Since

$$\alpha \propto \frac{e^2}{\hbar c} = \frac{e^2}{\varepsilon_p l_p} = \frac{m_e c}{\varepsilon_p l_p} = \frac{c}{L \varepsilon_p l_p}$$

Where L is the universe radius. Therefore,

$$\alpha \propto \frac{1}{L}.$$

As the universe expands, the fine structure constant (α) decreases. In addition, there is less interaction at a larger expansion of the universe. However, strong interaction occurs in the early universe, and it is possible that this accelerates the interaction of the early universe. We also found some structural objects in the early universe compared with the previous standard model.

CRediT authorship contribution statement

Chavis Srichan: Conceptualization, Formal analysis, Methodology, Writing – original draft, Writing – review & editing. **Pobporn Danvirutai:** Conceptualization, Formal analysis, Writing – original draft. **Adrian David Cheok:** Formal analysis, Supervision, Validation, Writing – review & editing. **Jun Cai:** Validation, Writing – review & editing. **Ying Yan:** Validation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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